Gaussian Pulse

- In Assignment A0, task 4.11.1 you are asked to define the initial displacement $\underline{u}_0$ as a "Gauss-modulated cosine". The cosine will give you the harmonic oscillations, while the Gaussian specifies the envelope (change in amplitude) of the cosine.

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Gaussian

coordinate, mean

$$e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

standard deviation

x : your coordinates

μ : mean – is the position of the peak

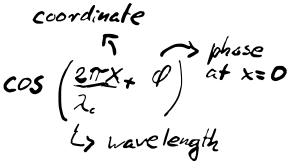
σ : standard deviation – a measure for the "width" of the Gaussian.

- Note: The Gaussian distribution is infinite in width, both in $x \to +\infty$ as well as $x \to -\infty$. This is why we need a statistical definition of the width (or equivalently via energy considerations). σ is sometimes also called the root-mean-square (RMS) width.

Gaussian function – Discussion

- Read the following references:
  - → Gaussian: https://en.wikipedia.org/wiki/Gaussian_function
  - → Standard deviation: https://en.wikipedia.org/wiki/Standard_deviation#Rules_for_normally_distributed_data
- Question: What is the peak height, i.e., amplitude, of the Gaussian on the previous slide?

Cosine



- The cosine is giving you the oscillations.
- After modulation, λ_c is the center wavelength. Consider that after modulation, the bandwidth of the signal is no longer zero. Instead your signal then contains a *spectrum* of wavelengths.
- The argument of the cosine is called the *phase*. Therefore,

$$\frac{2\pi L}{3\lambda_c} + \varphi \tag{1}$$

is the phase at $x = L/3$. According to the assignment, this phase should be zero.

Gauss-modulated cosine

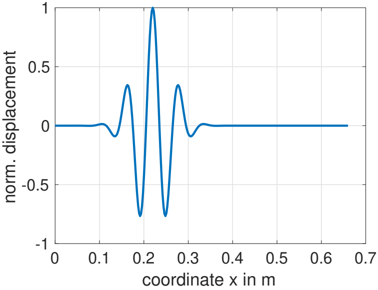
- Task description:
  → $\underline{u}_0$: This is the initial "disturbance". Use a Gauss-modulated cosine centered at $x_0 = L/3$, a 1-sigma width of $w = 4\,\mathrm{cm}$, and a center wavelength $\lambda_c = 6\,\mathrm{cm}$. Make sure that the phase of the cosine is zero at x_0. Write down the expression for $\underline{u}_0$ before implementing it.
- x_0 is giving you the "mean" of the Gaussian (its position).
- w is the standard deviation.
- λ_c is the wavelength.
- The orange sentence is giving you φ.
- Resulting Gaussian Pulse:

$$e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cos\left(\frac{2\pi}{\lambda_c} x + \varphi\right) \tag{2}$$

Lehrstuhl für Sensorik

Gaussian Pulse

- Resulting Gaussian pulse:



- Questions:
  - → Why do we need to specify an envelope?
  - → Why are we using a Gaussian pulse? Why not just a cosine modulated with a rectangular window, i.e., a burst with certain number of periods? If we did so, what would we need to change/consider in our FEM simulation?
  - → Perform the simulation for a 3-period burst!