Assignment 4: Nonlinear Magnetics – Inductive Displacement Sensor

Figure. 1 depicts an inductive displacement sensor. The shown axis-symmetric setup consists of a stationary iron yoke with a movable armature (also made of iron) at its center axis. A cylindrical copper coil with 200 windings is located inside the stationary inner part of the yoke and has an inner radius of $d_1 + d_2$ and an outer radius of $d_1 + d_2 + d_c$. Between the armature and the stationary parts of the yoke there is an air gap of height s which will strongly affect the inductance $L(s)$ of the overall assembly. This dependency can be exploited in practice to determine s from the measurable value $L(s)$ – as it was investigated for a similar setup during the winter term. This time, however, we want to consider a more realistic non-linear material behavior for the iron yoke and the iron armature.
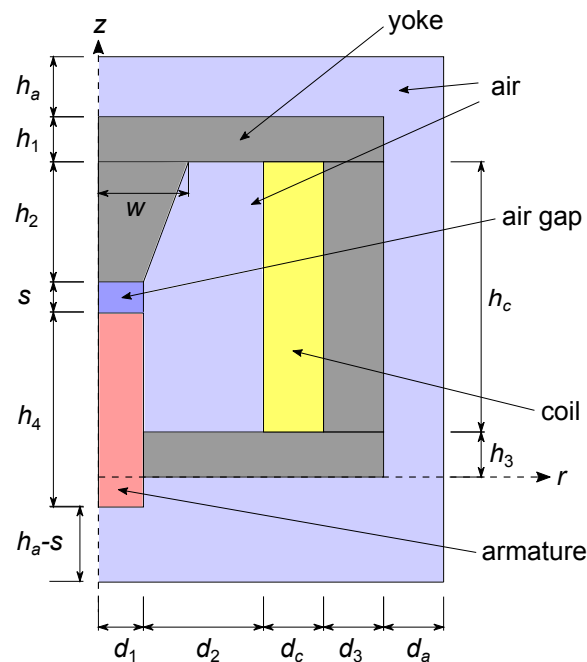

Figure 1: Inductive displacement sensor

General hints and remarks

- You should always correctly **label** the axes of the plots and add a **legend** if necessary. Format plots so that they can be **read easily**.

- **Save plots** as a `.png`-file before proceeding to the next task, as you will need to submit these.

- Write your **short answers** to all questions to a `Results.pdf` file (label each answer with its task number). Include your plots in this file.

- If you are unsure on how to perform the tasks, refer to

  - COMSOL Tutorial
  - COMSOL Multiphysics Reference Manual

1 Preparation

1.1. The inductance of the setup shown in Fig. 1 can be estimated analytically as

$$L_{\text{analytic}}(s) = \frac{N^2}{R_{\text{m,air}}(s) + R_{\text{m,yoke}}(s)} \approx \frac{N^2}{R_{\text{m,air}}(s)}, \tag{1}$$

where N is the number of turns of the coil, $R_{\text{m,air}}$ and $R_{\text{m,yoke}}$ are the magnetic resistances of the air gap and the yoke, respectively. The magnetic resistance of the air gap can be calculated as

$$R_{\text{m,air}}(s) = \frac{s}{\mu_0 \mu_r A}, \tag{2}$$

which uses the permeability in vacuum μ_0, the relative permeability of air μ_r and the cross-sectional area A of the air gap.

1.1.1. Why is it appropriate to neglect the resistivity of the yoke in (1)?

1.1.2. Is the curve $L_{\text{analytic}}(s)$ a line, a parabola, a hyperbola or an ellipse?

1.1.3. According to your last answer, is the inductive displacement sensor most appropriate to measure big or small displacements s? Why? Tip: consider the sensitivity of the sensor.

2 Modeling linear magnetics

In this section you will model a linear and stationary magnetic field. In the next section you will extend this model to consider a nonlinear relation between the magnetic field $\vec{H}$ and the magnetic flux density B.

2.1. Create a new 2D axi-symmetric model in Comsol. Add a *Magnetic Fields (mf)* physics as well as a *Stationary* study.

2.2. Define all required parameters with the following values:

- d1 = 5 mm, d2 = 7 mm, d3 = 8 mm, da = 15 mm, dc = 8 mm
- h1 = 8 mm, h2 = 20 mm, h3 = 8 mm, h4 = 33 mm, ha = 15 mm, hc = 45 mm
- w = 8 mm
- air gap: ss = 0.5 mm. You can't use s because it is a reserved variable name.
- element size: h_elem = 0.5 mm
- coil current: Iext = 1 A
- windings of the coil: N = 200

2.3. Model the geometry depicted in Fig. 1. Use a *Polygon* and a *Difference* node where appropriate.

2.4. Add *Air*, *Copper* and *Iron* from the materials library.

2.5. Set up the magnetic field to be simulated:

2.5.1. Inspect the node called *Ampère's Law*.
- How many material parameters are required? Which ones are those?
- Is the default setup a linear or a nonlinear simulation? Why?

2.5.2. Add appropriate boundary conditions (if required).

2.5.3. Model the excitation by imposing the current density $J = \frac{I_{\text{ext}} N}{d_c h_c}$ through the coil. We are assuming a filling factor of 100 %.

2.6. Create a free triangular mesh with maximum element size h_elem = 0.5 mm.

2.7. Add a *Parametric Sweep* to your stationary study. Sweep the coil current I_{ext} between 0 A and 5 A in steps of 1 A.

2.8. Perform the simulation. This should take just a few seconds.

2.9. Inspect the default plot of the magnetic flux density distribution for the different values of the parameter I_{ext}. How does the magnetic field change? Should this result be expected?

2.10. Investigate more closely on how the magnetic flux density changes with exciting current by plotting its magnitude at the upper center point of the armature, i.e., in the air gap, for different values of I_{ext}. What kind of behavior does the graph exhibit (constant, linear, quadratic, hyperbolic, exponential, etc.)? Why?

3 Modeling the nonlinear magnetic field

In order to obtain a more realistic model, in this section you will extend the simulation to consider a nonlinear magnetic material behavior.

The magnetic field can be described with two field variables, namely the magnetic field intensity $\vec{H}$ and the magnetic flux density $\vec{B}$. The Maxwell equations describe the time-space behavior of each of these two variables and they are *physical laws* that fulfill the purpose of balancing conserved quantities. However, in order to solve these equations, a relationship between $\vec{H}$ and $\vec{B}$ is needed in addition – the so called *constitutive equations* or *material law*. While the Maxwell equations can be derive by rigorous Gedankenexperiments (see Oxford English Dictionary and Encyclopædia Britannica), the material law – for the best of our knowledge – cannot.

For this reason, in practice it is necessary to find a "good" material law on empirical grounds. A good constitutive equation is characterized by an easy mathematical model and good computational results as compared to measurements. The first approach is, therefore, a linear relationship between $\vec{H}$ and $\vec{B}$. As we know from Taylor expansion theory, it is always reasonable to assume a linear behavior when the amplitudes, in this case of $\vec{H}$ and $\vec{B}$, are small enough. This is the kind of constitutive equation that was used in the previous section by imposing the permeability as linear coefficient between the two quantities. However, when large field amplitudes are involved, the linear approximation will not be appropriate anymore and a different material law has to be thought of. In the following, we will prescribe a material law based on measurements of both field variables which have been performed on a specific iron specimen.

Empirically, materials can be classified as *magnetically hard* and *magnetically soft*. The former exhibit a very pronounced hysteresis in their constitutive relation while the latter do not. For the following task, you will model a magnetically soft material by ignoring the hysteresis. This equivalently means that you will consider a purely *reversible process*, i.e., all of the magnetic energy can be converted back to other forms of energy. The magnetic energy density stored in the material is defined as

$$W_{\text{m}}(B') = \int_{0}^{B'} H(B)\,\mathrm{d}B, \tag{3}$$

while the so-called *magnetic coenergy density* is defined as

$$W_{\text{m}}'(H') = \int_{0}^{H'} B(H)\,\mathrm{d}H. \tag{4}$$

In the above equations, H and B denote the magnitudes of $\vec{H}$ and $\vec{B}$, respectively. It has also been assumed that $\vec{H}$ and $\vec{B}$ are in the same direction (a good approximation for most materials).

The magnetic energy density can be though of as the area above the B-H-curve (B on the y-axis, H on the x-axis) from $B = 0$ to $B = B'$. Correspondingly, the coenergy density would be the area below this curve from $H = 0$ to $H = H'$. Hence, the two are related to each other by

$$W_{\text{m}} + W_{\text{m}}' = H'B'. \tag{5}$$

Tasks:

3.1. Visualize W_m, W_m' and $H'B'$ in a linear B-H-curve for an arbitrary point (H', B') on the curve. Are W_m and W_m' equal?

3.2. For the linear B-H-curve, derive an expression for W_m which depends only on B' and H', i.e., avoids the integration.

3.3. Visualize W_m, W_m' and $H'B'$ in a nonlinear B-H-curve going into saturation for high values of H. Consider any arbitrary point (H', B') on the curve. Are W_m and W_m' equal?

In order to model a magnetically soft nonlinear material, Comsol needs to know how to compute three quantities: the B-H-curve, the inverse of the B-H-curve and the magnetic coenergy density as defined in (4). Comsol provides an easy mechanism to define these three. You can utilize this mechanism by performing the following steps:

3.4. Right click the *Materials* node in your component and select *Blank Material*.

3.5. Select the newly created node and label it "Iron nonlinear".

3.6. Disclose the *Material Properties* section and navigate to *Electromagnetic Models*. Right click on *B-H Curve* and select *Add to Material*.

3.7. Select the newly created node labeled *B-H* curve and inspect its settings. Note that the material model has two inputs: the current values $\vec{H}'$ and $\vec{B}'$ of the field variables. Their magnitudes are defined as *Local Properties* for your convenience.

3.8. In the *Output Properties* section the three required properties are already listed. Only their expression still needs to be defined. If the input to the model is $\vec{H}'$, the we would like to output $B(H')$. For this you need to prescribe the B-H-curve according to the measurements provided in the `bh1.txt` file provided on StudOn:

   3.8.1. Right click on the *B-H curve* node and select *Functions→Interpolation*.

   3.8.2. Select the newly created node and set the *Function name* to be `bh`.

   3.8.3. Import the values for the two columns `t` (corresponds to H) and `f(t)` (corresponds to B) from `bh1.txt` by clicking on the folder icon.

   3.8.4. Choose a linear interpolation between the measurement points.

   3.8.5. Chose a linear extrapolation below and above the measurement range. Set correct SI-units for the arguments (H) and the function values (B).

   3.8.6. Disclose the section *Related Functions*. Define the inverse function and label it `bh_inv`. Also define the primitive function and set its name to `bh_prim`.
   Note: the primitive function is also called an "indefinite integral" or "antiderivative", i.e., the primitive function of $f(t)$ is $F(t) = \int f(t)\, \mathrm{d}t$. Setting appropriate integration bounds, `bh_prim` defines the magnetic coenergy density, see (4).

   3.8.7. Click on *Plot* at the very top of the settings window to visualize the B-H-curve. Make sure that the curve drives into saturation for high values of H, as it would be expected.

3.9. After defining the B-H-curve as an interpolation between the measurement points, go back to the *B-H curve* node inside your materials node. Set the expressions for the output properties as follows:

   3.9.1. `normB`: `bh(normHin)`

   3.9.2. `normH`: `bh_inv(normBin)`

   3.9.3. `Wpm`: `bh_prim(normHin)`

3.10. Your magnetic material behavior is now ready. Select the top node of your material and apply the material to the yoke and armature domains. As the *Magnetic Fields* physics interface in Comsol implements a formulation that considers also the electrical field (remember that a time-varying magnetic field results in an electrical field), you have to specify additional electrical material properties for your model. You can do this in the following way:

3.10.1. Disclose the *Material Contents* section. Specify the *Electrical conductivity* to be $1 \times 10^7\,\mathrm{S/m}$ and the *Relative permittivity* as 1. The relative permeability is not needed for the non-linear simulation. In order to be able to use this material with linear simulations, specify the *Relative permeability* to be 4000.

Done! You now have a material with nonlinear magnetic behavior. However, this material model is not yet used for solving the magnetic field equations. The equations to be solved, including the constitutive equations, are defined by the *Ampère's Law* node in your *Magnetic Fields* physics.

3.11. Select the *Ampère's Law* node. Note that the selected constitutive relation is a relative permeability formulation, which is for linear magnetics only. You should keep this model for the coil and air domains.

3.12. To change to the nonlinear *B-H*-curve model in the yoke and the armature, right click on the *Magnetic Fields* node and selecte *Ampère's Law*. A second such node appears and overrides the first one on the domains it is defined on. Select only the yoke and armature domains for this node. Under *Constitutive Relation B-H*, select *B-H cuve* as *Magnetization model*. Both the magnetic field norm and the magnetic coenergy density have been defined in the material you created and should be taken from it. Also the electrical material parameters should be taken from the material node.

4 Analyzing the nonlinear magnetic field

4.1. Re-run the stationary study. This should take less than a minute.

4.2. Inspect the default plot showing the distribution of the magnetic flux density. Did it change compared to the previously performed linear simulation performed previously? If so, for what values of the exciting current I_ext do you get the biggest differences?

4.3. Look at the plot you created in task 2.10. Did the shape of the curve change? Why does it exhibit the behavior you see? Motivate your answer with the shape of your *B-H*-curve.

It is more instructive to plot both the linear an the nonlinear case into one graph. Do the following to prevent Comsol to override your simulation results:

4.4. Rename the *Study 1* node to *Study 1 - nonlinear*

4.5. Add a second stationary study and name it *Study 2 - linear*. Add the same *Parametric sweep* as in the first study.

4.6. Select the *Step 1: Stationary* node. In the *Physics and Variables Selection* section check the *Modify model configuration for study step* checkbox. Now disable the *Ampère's Law 2* node in the tree that appears. The default *Ampère's Law 1* node will then be used instead, which specifies a linear material behavior. This disables the nonlinear simulation for only this study.

4.7. Compute the new study to get the linear simulation results. The nonlinear simulation results from *Study 1 - nonlinear* remain and can still be used.

4.8. Your plot created in task 2.10. has been updated to show the results of the nonlinear simulation. Add a second curve to this plot showing the result of *Study 2 - linear*.

4.9. Add an appropriate legend to the plot.

4.10. The linear model can be considered a first order Taylor expansion of the nonlinear model.

4.10.1. According to your plot, at what operating point (H', B') has this Taylor expansion been performed?

4.10.2. In what range of exciting current I_ext could the linear model be used with good approximation?

4.10.3. If you were to be interested in the small-signal behavior around $I_\text{ext} = 2\,\mathrm{A}$, how would the relative permeability of the corresponding linear model change? I.e., would it be larger or smaller? Why?

5 The sensor response curve

Compute the response of the sensor to the displacement s for different driving currents I_{ext}.
Important: You should only use *Study 1 - nonlinear* for this task.

5.1. Add the parameter `ss` to the *Parametric sweep*. Sweep it from $0.5\,\text{mm}$ to $6\,\text{mm}$ with steps of $0.5\,\text{mm}$. Choose *All combinations* as *Sweep type*. Change the sweep values of I_{ext} to $1\,\text{A}$, $3\,\text{A}$ and $5\,\text{A}$.

5.2. Re-compute *Study 1 - nonlinear*. This might take about $10\,\text{min}$.

5.3. Plot the flux density magnitude at the upper center point of the armature in dependence of the displacement s for each of the exciting currents I_{ext}, i.e., create a figure with three curves, one for each value of I_{ext}.

    5.3.1. Does the maximal flux density increase or decrease with s? Why is this the case? Motivate your answer with (2).

    5.3.2. For which values of s would you expect the biggest influence of the nonlinear material?

5.4. Plot the inductance $L(s) = \frac{2E_{\text{m}}}{I_{\text{ext}}^2}$ (E_{m} is the total magnetic energy) for each of the three values of I_{ext}. For this you should create a *1D Plot Group* and add a *Global* node. Add an appropriate legend.

    5.4.1. Does this plot confirm your conjecture form task 5.3.2.?

    5.4.2. Which curve (i.e., for $I_{\text{ext}} = 1\,\text{A}$ or $I_{\text{ext}} = 3\,\text{A}$ or $I_{\text{ext}} = 5\,\text{A}$) is closest to the shape expected in task 1.1.2.?

    5.4.3. Does the sensitivity of the sensor tend to be larger or smaller when considering the nonlinear material behavior?

    5.4.4. Would you recommend using high or low driving currents for the sensor?

    5.4.5. What measurement range in the displacement s would you recommend for the sensor?

Submitting your assignment

Hand in your

- Comsol `.mph` file and

- a `Results.pdf` file with short answers to all questions within this assignment sheet (label each answer with its task number). All plots you are asked to create should also be contained in this file at the position of the corresponding task. Animations should be reduced to just one frame at a "representative" time. You can, for example, use Word or LaTeX to produce the PDF file.

Upload the above files to the corresponding Assignment section on StudOn:
https://www.studon.fau.de/exc2992793.html.

Important: Make sure to delete all results and meshes from the Comsol file before handing in. This reduces the file size. Refer to the Comsol Tutorial on how to do this.