

## Assignment 3: Magneto-mechanical Coupling – Skin effect and Lorentz force

This assignment deals with eddy-currents, Lorentz forces and their effects on magnetic and mechanical fields. These effects can either be wanted, like in the case of eddy current brakes, or can be unwanted, e.g., in the case of eddy current losses.

The axi-symmetric setup to be investigated is shown in Fig. 1. It consists of an iron yoke, a copper coil and a probe made of different materials. The inner space between yoke and probe is filled with air. An outer air region surrounding the setup can be neglected as the magnetic flux is assumed to be concentrated inside the yoke.

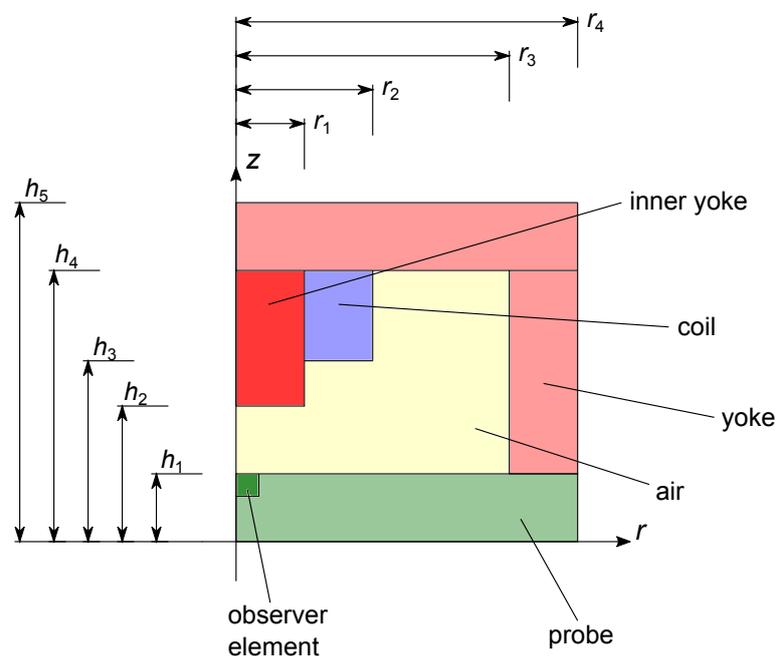


Figure 1: Magneto-mechanical setup used to investigate eddy currents and Lorentz forces.

### General hints and remarks

- You should always correctly **label** the axes of the plots and add a **legend** if necessary. Format plots so that they can be **read easily**.
- **Save plots** as a `.png`-file before proceeding to the next task, as you will need to submit these.
- Write your **short answers** to all questions to a `Results.pdf` file (label each answer with its task number). Include your plots in this file.
- If you are unsure on how to perform the tasks, refer to
  - [COMSOL Tutorial](#)
  - [COMSOL Multiphysics Reference Manual](#)

## 1 Preparation

- 1.1. Write down the formula for the penetration depth of eddy currents.
- 1.2. Using the above formula, compute the penetration depth in aluminum at 1 Hz, 10 Hz and 100 Hz.
- 1.3. State Lenz' law, which predicts the direction of induced eddy currents. What direction should be expected for the given setup if the coil current is in  $+\vec{e}_\varphi$ ?
- 1.4. What is the current density  $\vec{J}$  in the coil in dependence on the coil current  $I$ , the number of turns  $N$ , the coil's cross sectional area  $A$  (in the  $r$ - $z$ -plane) and the filling factor  $\eta$ ?
- 1.5. Provided you have some complex variable  $c = \text{Re } c + i \text{Im } c$ , how can you compute its phase angle?

You will first perform a harmonic magnetic simulation to investigate on eddy currents. Only then will the electromagnetic field be coupled to mechanics.

## 2 Modeling the magnetic field

- 2.1. Create a axi-symmetric Comsol model with a *Magnetic Fields (mf)* physics and a *Frequency Domain* study. How is the *Magnetic Fields, No Currents (mfnc)* physics interface – which is also offered in Comsol – different to the *Magnetic Fields (mf)* physics?
- 2.2. Define the following parameters:
  - $r_1 = 10$  mm,  $r_2 = 15$  mm,  $r_3 = 30$  mm,  $r_4 = 40$  mm
  - $h_1 = 50$  mm,  $h_2 = h_1 + 5$  mm,  $h_3 = h_1 + 8$  mm,  $h_4 = h_1 + 18$  mm,  $h_5 = h_1 + 28$  mm
  - Element size  $h_{\text{mesh}} = 0.4$  mm
  - Coil current  $I_{\text{ext}} = 1$  A
  - the coil cross sectional area  $A_{\text{coil}}$  according to the values given above
  - the coil filling factor  $\eta = 0.9$
  - number of coil turns  $N = 100$ .
- 2.3. Model the geometry as depicted in Fig. 1. Make sure that the “air” domain is not a rectangle. It should rather have the geometry as seen in the figure. You can achieve this by first modeling a rectangle and then using a *Difference* node to “cut out” the areas that you do not need.

For easier selection of domains, you will use the *cumulative selections* feature:

- 2.4. Go back to each of the nodes modeling your geometry. At the bottom of the settings window you can specify which cumulative selection this entity should contribute to. Create four cumulative selections named *probe*, *air*, *yoke* and *coil*. Add each of the geometric entities to its appropriate cumulative selection.
- 2.5. After building the geometry, you will find the *Cumulative Selections* at the very bottom of the *Geometry node*. Inspect them to make sure that your cumulative selections are correct.
- 2.6. Add *Air*, *Iron*, *Aluminum* and *Copper* as materials. Make *Air* the default material (first node in the tree).
- 2.7. Select the *Iron* node. In the *Geometric Entity Selection* section of the settings window, set the selection to the cumulative selection named “yoke”. This will select the geometric entities as defined by the cumulative selection created in 2.4..
- 2.8. In a similar way, specify the probe to be made out of aluminum and the coil out of copper.

2.9. Generate a *Free Quad* mesh with element size specified by the parameter  $h_{\text{mesh}}$ .

Now, set up the physics and the harmonic simulation:

2.10. Set appropriate boundary conditions (if needed).

2.11. Define the excitation using an *External Current Density domain* node. The selection should be *coil* (the cumulative selection you defined). Specify the current density in  $+\vec{e}_\varphi$  as computed in task 1.4.

2.12. Perform the simulation for 1 Hz, 10 Hz and 100 Hz.

### 3 Analyzing the resulting eddy currents

To understand the formation of eddy currents, analyze the results with the plots described in the following:

3.1. Create an *Arrow Surface* plot that shows the magnetic flux density at  $80 \times 150$  grid points with logarithmic arrow length. Scale the arrows appropriately. Note that the magnetic flux density forms eddies, highly dependent on frequency. How is this possible? Would you be able to observe this behavior with a magnetostatic simulation? Add the magnetic streamlines in the same graph to get an even better visualization.

3.2. Make surface plots of the  $\varphi$ -component of the current density excluding the coil region showing:

3.2.1. the absolute value (this gives the local magnitude of the current density but neither the direction in which the current flows nor its phase with respect to the excitation)

Are your plots in agreement with the penetration depths computed in task 1.2.?

3.2.2. the real part

3.2.3. the imaginary part

Comparing the imaginary and real parts, what value would you expect the phase of  $\vec{J}$  to be when considering the surface of the probe?

3.3. Note that the magnetic flux density  $\vec{B}$  is a direct consequence of the exciting coil current  $I_{\text{ext}}$ . The eddy current density  $\vec{J}$ , however, is proportional to the induced electric field intensity as  $\vec{J} = \sigma \vec{E} \sim -d\vec{B}(t)/dt$ . Therefore, in the harmonic case, a  $90^\circ$  phase shift could be expected.

3.3.1. Plot the phase of the eddy current component  $J_\varphi$  according to task 1.5. on the top line of the probe. The plot should contain one line for each frequency. Add an appropriate legend.

Why is the phase relative to the excitation always lower than  $90^\circ$ ? At which frequency do you expect the highest loss in energy (at least when considering the top line of the probe)?

3.4. Remember that the power density lost due to resistive losses (Joule heating) when the field variables vary with  $e^{i\omega t}$  is given by  $w_{\text{dissipated}} = \frac{1}{2} \vec{J}^* \cdot \vec{E}$  (analogous to  $P = \frac{1}{2} I^* U$ ), where  $\bullet^*$  stands for the complex conjugate.

3.4.1. Make a surface plot showing  $w_{\text{dissipated}}$ .

3.5. The induced eddy currents interact with the magnetic flux density resulting in so-called *Lorentz forces*. These forces can be computed with the electromagnetic field without the need of a mechanical simulation. Create a surface plot showing the time-averaged Lorentz force density in  $z$ -direction in the probe. In the same graph, also plot the Lorentz force vectors with an *Arrow Surface* plot.

This force is used in the next section to drive a mechanical simulation of the setup.

## 4 Adding mechanics

You will now additionally consider the mechanical field. To model the mechanics:

- 4.1. Add the *Solid Mechanics (solid)* physics to the *Component 1*. Apply it to all solid components.
- 4.2. Fix the right boundary of the setup, i.e., the cylinders are clamped.

The solid mechanics and the electromagnetic fields still need to be coupled:

- 4.3. Right click on the *Multiphysics* node and choose *Lorentz Coupling*.
- 4.4. In the settings window of the new node, make sure to couple the correct fields.
- 4.5. Inspect the coupling equations. Is this a strong or a weak coupling? Will Comsol need to perform an iterative solution procedure or would a direct method work?  
Consider a fixed setup which is not able to move, i.e., the velocity  $\mathbf{v} = \mathbf{0}$ . In this case, type of coupling change? Which one would you have now? Why?

Now you are ready to perform the coupled magneto-mechanical simulation.

- 4.6. Perform the harmonic analysis for the same frequencies as before.
- 4.7. Re-inspect the plots you have created previously. Are changes visible in the plots?
- 4.8. Make a *Surface plot* showing the von Mises stresses. Add a *Deformation* node to it.
- 4.9. Plot the  $z$ -displacement of the top edge of the probe over the  $r$ -coordinate.

The results show that the probe is much stiffer than the yoke. Let's consider a disc:

- 4.10. Change  $h_1$  to 1.2 mm.
- 4.11. Re-compute the simulation.
- 4.12. Re-inspect the plots visualizing the mechanical field.

## 5 Transient simulation

Lastly, you will perform a transient simulation.

- 5.1. Change the *External Current Density* to a sinusoidal burst at 50 Hz with 3 periods. Define any parameters you may need.
- 5.2. Add a *Time Dependent* study to your model.
- 5.3. The total duration of the simulation should be 20% longer than your excitation. Discretize every period of your excitation with 30 time steps. Make sure that the time stepping is set to *strict*.
- 5.4. Perform the simulation. This should take less than a minute.
- 5.5. Create a 1D Plot Group which shows the following quantities over time at the center top point of the probe:
  - the  $z$ -displacement normalized to its maximum
  - the  $z$ -component of the magnetic flux density normalized to its maximum
  - the  $\varphi$ -component of the current density normalized to its maximum

Add an appropriate legend.

- 5.5.1. At what frequency does the plate vibrate? Why is this the case? To answer this, inspect how the Lorentz forces  $\mathbf{F}_v$  are computed (see the *Lorentz Coupling* node).
- 5.5.2. Is the transformation from electromagnetic to mechanical energy a linear or a nonlinear effect?

## Submitting your assignment

Hand in your

- Comsol .mph file and
- a `Results.pdf` file with short answers to all questions within this assignment sheet (label each answer with its task number). All plots you are asked to create should also be contained in this file at the position of the corresponding task. Animations should be reduced to just one frame at a “representative” time. You can, for example, use Word or LaTeX to produce the PDF file.

Upload the above files to the corresponding Assignment section on StudOn:  
<https://www.studon.fau.de/exc2992793.html>.

**Important:** Make sure to delete all results and meshes from the Comsol file before handing in. This reduces the file size. Refer to the Comsol Tutorial on how to do this.