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CAE of Sensors and Actuators
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Definitions

- u, v: scalar fields, $\vec{u}, \vec{v}$: vector fields
- a, b: real numbers describing the interval $[a, b]$
- $\bullet' = \frac{\partial \bullet}{\partial x}$
- Ω: domain, Γ: surface
- $\partial\Omega$: surface enclosing the domain Ω
- $\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right]^T$
- $\vec{n}$: unit normal vector on the surface $\partial\Omega$, depends on the position $\vec{x} \in \partial\Omega$

1D Integration by Parts

(1D version of Greens integral theorem)

$$\int_a^b v u'' \,\mathrm{d}x = \left[v u'\right]_a^b - \int_a^b v' u' \,\mathrm{d}x \tag{1}$$

Greens Integral Theorem

$$\int_\Omega v \nabla \cdot \nabla u \,\mathrm{d}\Omega = \oint_{\partial\Omega} v \nabla u \cdot \vec{n} \,\mathrm{d}\Gamma - \int_\Omega \nabla v \cdot \nabla u \,\mathrm{d}\Omega \,, \tag{2}$$

Greens Integral Theorem in Vector Form

$$\int_\Omega \vec{v} \cdot \nabla \times (\nabla \times \vec{u}) \,\mathrm{d}\Omega = \int_\Omega (\nabla \times \vec{v}) \cdot (\nabla \times \vec{u}) \,\mathrm{d}\Omega - \oint_{\partial\Omega} (\vec{v} \times \nabla \times \vec{u}) \cdot \vec{n} \,\mathrm{d}\Gamma \tag{3}$$

Integral Theorem of Gauss

$$\int_\Omega \nabla \cdot \vec{u} \,\mathrm{d}\Omega = \oint_{\partial\Omega} \vec{u} \cdot \vec{n} \,\mathrm{d}\Gamma \tag{4}$$

Integral Theorem of Stokes

$$\int_\Gamma \nabla \times \vec{u} \cdot \vec{n} \,\mathrm{d}\Gamma = \oint_{C(\Gamma)} \vec{u} \cdot \vec{t} \,\mathrm{d}s \,, \tag{5}$$

where:
$C(\Gamma)$: closed contour of the surface Γ
s: line
$\vec{t}$: tangential vector to the contour $C(\Gamma)$, depends on the position $\vec{x} \in C(\Gamma)$