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CAE of Sensors and Actuators
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Exercise sheet 1

In the following we denote with u the scalar dependent variable, x, y, z shall be the independent variables, u' the derivative with respect to x and f is the excitation. Furthermore, $\ddot{u}$ denotes the second time derivative of u, c is a constant and Ω is the domain of the boundary value problem (BVP). The boundary of the domain is $\partial\Omega$ and it splits into the Dirichlet boundary Γ_D and the Neumann boundary Γ_N such that $\Gamma_D \cup \Gamma_N = \partial\Omega$ and $\Gamma_D \cap \Gamma_N = \emptyset$.

Given the strong form, derive the corresponding weak form of the following boundary value problems:

1. Example 1

$$\begin{aligned} &u'' = f \quad \text{on } x \in \Omega = (a, b) \\ &u(a) = g_a \\ &u(b) = g_b \end{aligned}$$

2. Example 2

$$\begin{aligned} &-u'' + cu = f \quad \text{on } x \in \Omega = (a, b) \\ &u(a) = g_a \\ &u'|_b = h_b \end{aligned}$$

3. Example 3

$$\begin{aligned} &-u'' + cu' = f \quad \text{on } x \in \Omega = (a, b) \\ &u(a) = g_a \\ &u(b) = g_b \end{aligned}$$

4. Example 4

$$\begin{aligned} &-\ddot{u} + u'' + cu = f \quad \text{on } x \in \Omega = (a, b) \\ &u(a) = g_a \\ &u(b) = g_b \end{aligned}$$

5. Example 5

$$\begin{aligned} &-\Delta u + cu = f \quad \text{on } \vec{x} = (x, y)^T \in \Omega \\ &u(x, y)|_{\Gamma_D} = g_D \\ &\left.\frac{\partial u}{\partial \vec{n}}\right|_{\Gamma_N} = \nabla u \cdot \vec{n}|_{\Gamma_N} = h_N \end{aligned}$$

Solutions

Using test function v:

1. Example 1

$$-\int_a^b v'u'\,\mathrm{d}x = \int_a^b vf\,\mathrm{d}x$$
$$\text{with DBC: } u(a) = g_a\,, \quad u(b) = g_b$$

2. Example 2

$$\int_a^b v'u'\,\mathrm{d}x + \int_a^b cvu\,\mathrm{d}x = \int_a^b vf\,\mathrm{d}x + v(b)\,h_b$$
$$\text{with DBC: } u(a) = g_a$$

3. Example 3

$$\int_a^b v'u'\,\mathrm{d}x + \int_a^b cvu'\,\mathrm{d}x = \int_a^b vf\,\mathrm{d}x$$
$$\text{with DBC: } u(a) = g_a\,, \quad u(b) = g_b$$

4. Example 4

$$-\int_a^b v\ddot{u}\,\mathrm{d}x - \int_a^b v'u'\,\mathrm{d}x + \int_a^b cvu\,\mathrm{d}x = \int_a^b vf\,\mathrm{d}x$$
$$\text{with DBC: } u(a) = g_a\,, \quad u(b) = g_b$$

5. Example 5

$$\int_\Omega \nabla v \cdot \nabla u\,\mathrm{d}\Omega + \int_\Omega cvu\,\mathrm{d}\Omega = \int_\Omega vf\,\mathrm{d}\Omega + \int_{\Gamma_N} v\,h_N\,\mathrm{d}\Gamma$$
$$\text{with DBC: } u(x,y)|_{\Gamma_D} = g_D$$