

Assignment 4: Magnetic Field – Inductive Positioning Sensor

Figure 1 depicts the **axis-symmetric** setup of an inductive positioning sensor. It consists of a copper coil of length l , inner radius r and outer radius $r + w$ and a movable iron core of length l and radius r_{Core} . The whole assembly operates in air.

The working principle of such kind of sensors is to measure the inductance $L(s)$ of the coil depending on the offset s between coil and iron core. Once the relation between s and $L(s)$ is known, the quantity of interest s can be calculated from the measurable value $L(s)$.

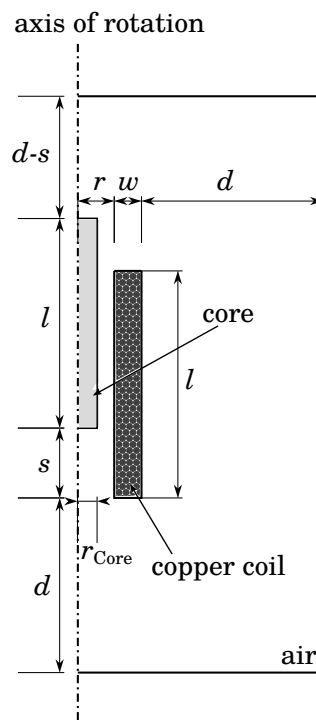


Figure 1: Inductive positioning sensor.

General hints and remarks

Please make sure to:

- Save all relevant plots you generate (e.g., as png-file) while working on the assignment.
- Always label the axes of the plots and add a legend if necessary.
- Format your plots so that they can be read easily (e.g., appropriate font size).
- Prepare a **Results.pdf** file with the plots and short answers to all questions (label each answer with its task number) for submission.
- If you are unsure on how to perform the tasks, refer to
 - [Comsol Tutorial: Electrostatic Field](#)
 - [COMSOL Multiphysics Reference Manual](#)

1 Preparation

- 1.1. Estimate the inductance L of the setup using the analytic formula for long air filled solenoids:

$$L_{\text{analytic}} = \mu_0 \frac{N^2 r^2 \pi}{l}. \quad (1)$$

Assume thereby that all parameters assume the same values as in tasks 2.2. and 2.5.2.).

- 1.2. If the coil is loaded with a current I , the total stored magnetic energy is given by

$$W = \frac{1}{2} L I^2. \quad (2)$$

Compute the stored energy of the inductance L_{analytic} (obtained in task 1.1.) if it is loaded with $I = 1$ mA.

2 Modeling

- 2.1. Create a new COMSOL Project with the *Model Wizard*. Choose the appropriate geometry type. Add a *AC/DC* → *Magnetic Field (mf)* physics node together with a *Stationary* study type.

- 2.2. Model the **geometry** as shown in Fig. 1. Define and use the following parameters with their corresponding values:

- length $l = 8$ cm of coil and core,
- inner radius $r = 0.9$ cm of the coil,
- width $w = 0.2$ cm of the coil,
- position offset $s = 0$ cm (make sure it works with non-zero values as well),
- radius $r_{\text{core}} = 0.75$ cm,
- boundary $d = 16$ cm (this is quite large to allow the core to move).

Note: make sure not to name the parameters with reserved names (COMSOL will highlight them in a yellow color).

- 2.3. Set up the **materials** in each of the domains.

- 2.4. Set up the **physics** node:

2.4.1. Make sure that the magnetic field is calculated on the whole domain.

2.4.2. What is the *dependent variable* COMSOL is solving for? How is it related to the *magnetic flux density* B ?

2.4.3. Write down the equation that COMSOL solves into your **Results.pdf**. Thereby, replace all physical quantities into the main equation (Ampère's law), such that the equation depends only on the dependent variable, the material parameters (permeabilities μ_0 , μ_r , conductivity σ), as well as the excitation (electrical current density J). How many scalar equations are being solved by COMSOL?

2.4.4. What polynomial order of the shape functions is used by default in COMSOL? Make sure to use *quadratic shape functions*.

2.4.5. Set up appropriate boundary conditions. What is the default boundary condition? Is it a Neumann or a Dirichlet boundary condition in terms of the dependent variable? What does it imply for the magnetic flux density \vec{B} ?

- 2.5. The **excitation** should consist in loading the coil with $I = 1$ mA:

2.5.1. Define I as a parameter.

2.5.2. The coil shall consist of $N = 400$ turns and a filling factor of 1. Define N as a parameter.

- 2.5.3. Define a parameter representing the current density $J = NI/(lw)$ flowing through the modeled cross section of the coil. Use the previously defined parameters for this definition.
 - 2.5.4. Add an *External Current Density* domain contribution to your physics, apply the node to the correct domain and set the component J_ϕ of the imposed current density to the parameter from 2.5.3.
 - 2.5.5. When analyzing the inductance of the setup, would it make a difference if you were using another value for the loading current I ? Why?
- 2.6. **Mesh** the geometry using triangular elements of maximum element size $h_{\max} = 0.25$ cm.

3 Validation of the model

For this step, assume that the core is made out of **air**.

- 3.1. Deactivate the *Iron* material node and use air as material for the core.
- 3.2. Perform a *Stationary/Static* simulation. This should only take a few seconds.
- 3.3. The magnetic flux density magnitude is plotted as a surface plot per default. Add to it an *Arrow Surface* and plot the **normalized** magnetic flux density vectors $\vec{B}/|\vec{B}|$ (*Coloring and Style*→*Arrow length*→*Normalized*). How does the magnetic flux density \vec{B} behave at the boundaries? Did you expect that behavior? Why? Compare your result to task 2.4.5.
- 3.4. Is \vec{B} parallel or perpendicular to the *Contour* lines plotted per default by COMSOL?
- 3.5. Compute the resulting inductance $L_{\text{simulated}}$ using *Derived Values*→*Global Evaluation*. You can access the total stored magnetic energy in COMSOL. Use eq. (2) to obtain the simulated inductance $L_{\text{simulated}}$ from the simulated magnetic energy.
- 3.6. Compare $L_{\text{simulated}}$ to L_{analytic} obtained in task 1.1. Why is there a difference between the results? How large is the deviation in percent? Is the difference acceptable, e.g., a few percent? If not, adjust the simulation to better fit with your analytic estimate (e.g., increase discretization, increase simulation domain).

4 Analysis

For the actual sensor setup, an iron core will be considered.

- 4.1. Change the material of the core back to **iron**.
- 4.2. Perform a parametric sweep on the displacement s of the core with the following values: from 0 cm to 8 cm in steps of 0.5 cm.
- 4.3. Plot and inspect the magnetic flux density for the different values of s . Remark: You only need to save the plot for $s = 4$ cm. What can you observe regarding the magnetic flux density inside the shifted core? Is it drawn into the core or is it repelled? Why is that?
- 4.4. An inductor is able to store magnetic energy. Visualize the stored magnetic energy density for the different positions s of the core (save only for $s = 4$ cm). **Important:** make sure to set a manual color range of $0 \mu\text{J}/\text{m}^3$ to $200 \mu\text{J}/\text{m}^3$; that way all your plots (for different values of s) will have the same color range and are easily comparable. Where is the magnetic energy mainly stored (i.e., the core, the coil, the surrounding air)?
- 4.5. Plot the inductance L over the sensor displacement s .
- 4.6. Estimate the sensitivity S_L of the setup via a forward difference as follows:

$$S_L(s_i) = \frac{L_{\text{simulated}}(s_{i+1}) - L_{\text{simulated}}(s_i)}{s_{i+1} - s_i}, \quad (3)$$

where $L_{\text{simulated}}(s_i)$ is the simulated inductance corresponding to a position offset of $s = s_i$. Note that the array storing S_L will be one entry shorter than the one of $L_{\text{simulated}}$. For your plots consider the x -range for the sensitivity plot to go from 0 cm to 7.5 cm (dropping the value 8 cm). You might prefer to use MATLAB for this task.

- 4.7. Considering that a sensor should ideally show a linear relation between the input s and the output $L(s)$, what would be an appropriate measuring range of this sensor, i.e., for what values of s would you get a nearly linear behavior?

Submitting your assignment

Hand in your group's

- COMSOL `mph`-file (clear all meshes and results beforehand, see the COMSOL Tutorial), and
- `Results.pdf` file with the plots and short answers to all questions (label each answer with its task number),

before the due date by uploading to the corresponding StudOn section.

When handing in for the first time you need to create a group. Please add all members when doing so! Once the group has been create, you can and should re-use the group for the subsequent submissions.