Assignment 2: Electrostatic Field – Capacitive Sensor

The following assembly consists of a **cylindrical** capacitor with three aluminum electrodes. The space between the middle and the ground electrode is filled with the dielectric material strontium titanate ($SrTiO_3$). The remaining space is filled with air. Between the top and the ground electrode a constant voltage of V_0 is applied. The potential at the middle electrode is unknown but can be assumed to be uniform on the whole electrode. The dimensions of the capacitor can be seen in Fig. 1.
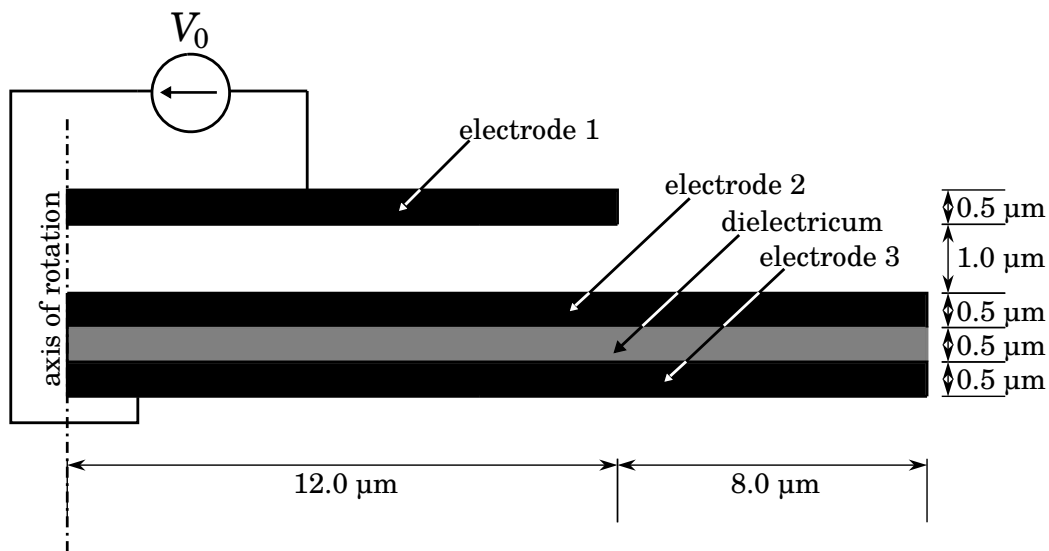

Figure 1: Cylindrical capacitor

General hints and remarks

Please make sure to:

- <u>Save</u> all relevant plots you generate (e.g., as `png`-file) while working on the assignment.

- Always <u>label the axes</u> of the plots and <u>add a legend</u> if necessary.

- Format your plots so that they can be <u>read easily</u> (e.g., appropriate font size).

- Prepare a `Results.pdf` file with the plots and <u>short</u> answers to all questions (label each answer with its task number) for submission.

- If you are unsure on how to perform the tasks, refer to

  - Comsol Tutorial: Electrostatic Field
  - COMSOL Multiphysics Reference Manual

1 Preparation

1.1. Derive an **analytic formula** to estimate the capacitance of the assembly shown in Fig. 1. You can assume that electrode 1 and electrode 2 form a plate capacitor of radius r_1 and electrode 2 and electrode 3 another plate capacitor of radius r_2. How do you have to connect these two capacitors and what is the resulting formula for the overall capacitance of the setup depending on r_1 and r_2?

1.2. Calculate the resulting capacitance of the setup if the relative permittivity of air is 1 and that of $SrTiO_3$ is 250.

2 Modeling

To simulate the capacitor in Fig. 1 in COMSOL Multiphysics:

2.1. Define all parameters you need in the `Parameters 1` node. This should include geometric parameters (see Fig. 1) as well as the element size parameters `elem_size`. Set the element size to `0.2` μm. Additionally add a parameter for the boundary size `h_boundary`.

2.2. Model the geometry as shown in Fig. 1. Each electrode should be modeled with its own `Rectangle`.

2.3. Add `Electrostatics (es)` as `Physics` and select the whole domain for simulation. Questions:

    2.3.1. What are the equations being solved for? What does each of the variables stand for? If you don't know: look up the `COMSOL Multiphysics Reference Manual`.

    2.3.2. What is the `Dependent Variable` and what is its default variable symbol?

    2.3.3. What is the default *discretization order* for this physics (i.e., the order of the ansatz functions)?

    Note: All of this can be found in the `Settings` window of the `Electrostatics (es)` node.

2.4. Add a `Blank Material` to the `Component 1` → `Materials` node. Name it `Air` and set the `Relative permittivity` to 1.

2.5. Add one more `Blank Material` to the `Component 1` → `Materials` node. Name it `SrTiO3` and set the `Relative permittivity` to 250.

2.6. Add and configure appropriate *boundary conditions* (BCs) and/or *constraints* to the `Electrostatics (es)` node. Questions:

    2.6.1. State all the BCs/constraints you used and how they relate to Dirichlet/Neumann BCs.

    2.6.2. How did you model each of the electrodes?

    2.6.3. What material did you assign to the electrodes? Is the material relevant in your model? If so, how does it influence the model?

    2.6.4. What alternatives could you use to model the electrodes in an electrostatic simulation with COMSOL Multiphysics?

2.7. Set the `Maximum element size` for the mesh to the parameter you defined as `elem_size`.

3 Analysis

Perform a `Stationary Study`. Note that this should not take more than a few seconds to compute.

3.1. Inspect the `Electric Potential (es)` surface plot. Is the solution plausible? If not, review and correct your **model**!

3.2. Play with the permittivity value of $SrTiO_3$. How does the solution change?

3.3. Create a new `2D Plot Group`. Add an `Arrow Surface` plot and plot the electric field intensity vectors $\vec{E}$. Configure the arrow positioning and scaling to obtain visually good results. Additionally add a `Contour` to visualize the *equipotential lines* in the same graph. Choose a nice `Color table` for the lines (e.g., `Cividis`). Read about the importance of *perceptually uniform colormaps* [2, 1].

3.4. Perform a mesh refinement study:

    3.4.1. Add a `Parametric Sweep` to your `Study` for the `elem_size` parameter with values between $0.05\,\mu\text{m}$ and $0.5\,\mu\text{m}$.

    3.4.2. Plot the `Total electric energy` vs. the number of degrees of freedom of the model. The latter quantity is available under `Global definitions` as the variable `numberofdofs`. How do the degrees of freedom relate to the number of elements, the element type and the type of ansatz functions?

    3.4.3. Create a new `2D Plot Group` with a `Surface` plot of the electric field intensity magnitude $|\vec{E}|$.

- Where does the maximum occur?
- How does the maximum change with `elem_size`? Note: You can display the maximum value by activating `Show maximum and minimum values` in the `Color Legend` settings of the `2D Plot Group`.

    3.4.4. Add a maximum selection to your data set: Right click on `Data Sets` and select `More Data Sets` $\rightarrow$ `Maximum`. In the settings of the new node select the parametric solutions as source of the data set. This `Maximum` node contains the maximum values of all the solutions in the parametric data set and can now be used for plotting.

    3.4.5. Create a `1D Plot Group` and add a `Point Graph`. Select `Maximum 1` as Data set for the plot. Plot `es.normE*elem_size^2` on the y-axis and `elem_size` on the x-axis. Does the specified quantity increase or decrease with reduced element size?

3.5. Based on your mesh refinement study, choose a value for `elem_size` which yields a converged solution. Now perform a parameter study on the boundary size (`h_boundary`) for this fixed value of the element size. Hint: avoid `h_boundary = 0`, as this might lead to problems due to ambiguous BCs.

3.6. Plot the capacitance (i.e., $2E/U^2$, with the total energy E and applied voltage U) over `h_boundary`. How big should the boundary be to ensure that most of the electric energy is captured inside the simulation domain? Perform all following tasks with an appropriate boundary.

3.7. Compare the numerically computed capacitance of the setup to your analytical estimate. What is the difference between the two? Why are they not the same?

3.8. Deactivate the previous Parametric sweeps and perform a parameter study on the distance between electrode 1 and electrode 2. Can the setup be used as a proximity sensor to metallic objects? Does the sensor show a linear behavior?

Submitting your assignment

Hand in your group's

- COMSOL `mph`-file (clear all meshes and results beforehand, see the COMSOL Tutorial), and

- `Results.pdf` file with the plots and <u>short</u> answers to all questions (label each answer with its task number),

before the due date by uploading to the corresponding StudOn section.

When handing in for the first time you need to <u>create a group</u>. Please add all members when doing so! Once the group has been create, you can and should <u>re-use it</u> for the subsequent submissions.

References

[1] Jamie R. Nuñez, Christopher R. Anderton and Ryan S. Renslow. "Optimizing colormaps with consideration for color vision deficiency to enable accurate interpretation of scientific data". en. In: *PLOS ONE* 13.7 (Jan. 2018), e0199239. DOI: 10.1371/journal.pone.0199239.

[2] Bob Rudis, Noam Ross and Simon Garnier. *The viridis color palettes*. Mar. 2018. URL: https://cran.r-project.org/web/packages/viridis/vignettes/intro-to-viridis.html.