Calculating the full leaky Lamb wave dispersion characteristics

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Motivation

Guided ultrasonic waves in solid media:

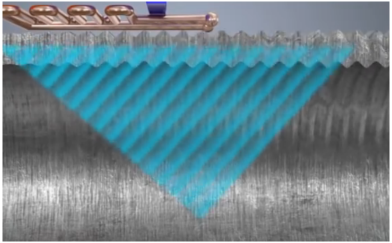


Figure: Wave inside the pipe wall[1]

- nondestructive testing
  - → long range inspection
  - → e. g. inspection of pipes
- ultrasonic sensing
  - → e. g. flow metering

How does the *interaction* between the plate and the adjacent fluid take place?

Lehrstuhl für Sensorik

[1] ROSEN Group. *ROSEN EMAT Flowmeter*. 2018. URL: http://flowmeter.rosen-group.com/

Content

Lehrstuhl für Sensorik

Leaky Lamb waves – Introduction

- **leaky Lamb waves:** guided waves inside a plate which is in contact with a fluid
- leakage of energy into the adjacent fluid
- many modes exist
- each with a unique displacement structure

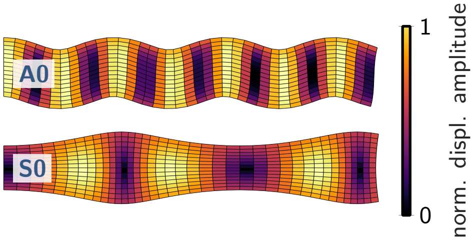


Figure: Basic modes in a **free** plate.
A0: anti-symmetric Mode. **S0**: symmetric Mode.

- They are dispersive:

Dispersion

wavenumber k_x depends on frequency f:

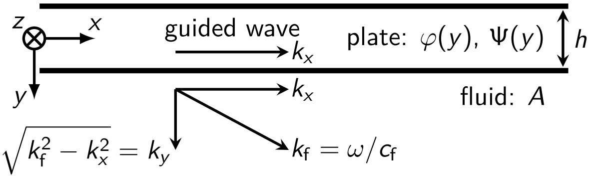
$$k_x = k_x(f) \quad \Leftrightarrow \quad c_p = c_p(f)$$

- The dependence is implicitly given by the **characteristic equation**:

$$F(k_x, f) = 0\,.$$

- It is transcendental!
  - $\rightarrow$ **numerical methods** are required
  - $\rightarrow$ badly conditioned problem!

Leaky Lamb waves – Modeling



- **plate**: assume harmonic wave propagation in x-direction:

$$\varphi_p(x, y, t) := \varphi(y)\, e^{i(k_x x - \omega t)}$$
$$\Psi_p(x, y, t) := i\Psi(y)\, e^{i(k_x x - \omega t)}$$

- **half-space** $y > h/2$ (fluid domain)
  - → open domain: exchange of energy
  - → assume plane harmonic wave:

$$\varphi_f(x, y, t) := A\, e^{i k_y y}\, e^{i(k_x x - \omega t)}$$

  - → scalar degree of freedom A

closed computational domain
$y \in [-h/2, h/2]$

In terms of the potentials $q = [\varphi(y), \Psi(y), A]^T$, write:

1. equations of motion
2. top boundary conditions
3. bottom interface conditions

- results in a

differential, nonlinear eigenvalue problem in k_x

exact fluid-structure interaction

Leaky Lamb wave problem

- to solve on a computer
  - → discretize the differential eigenvalue problem (e. g., spectral collocation, finite elements, finite differences)
  - → **algebraic** nonlinear eigenvalue problem

nonlinear eigenvalue problem: leaky Lamb waves

$$\underline{\underline{F}}(k_x)\underline{q} = \underline{0}\,, \quad \text{with} \quad \underline{\underline{F}}(k_x) = k_x^2\underline{\underline{A}}_2 + k_x\underline{\underline{A}}_1 + \underline{\underline{A}}_0 + \mathrm{i}\,k_y\underline{\underline{B}}\,,$$

$$\text{where } k_y = \sqrt{k_\mathrm{f}^2 - k_x^2}\,.$$

- the matrix function $\underline{\underline{F}}(k_x)$ is:

nonlinear	not unique	not holomorphic

⇒ difficulties when solving

Common approaches

- nonlinear eigenvalue solvers (e. g. iterative linearization, contour integration)
  - not always reliable [4], e. g., might not find all solutions.
  - no standard methods
- perfectly matched layer (PML)
  - additional discretization of fluid
  - spurious eigenvalues need to be discarded correctly
  - needs to be designed correctly

In the following:

Convert the nonlinear eigenvalue problem to a **linear** eigenvalue problem.

+ use modern linear eigenvalue solvers

[4] Volker Mehrmann and Heinrich Voss. "Nonlinear eigenvalue problems: a challenge for modern eigenvalue methods". en. In: *GAMM-Mitteilungen* 27.2 (Dec. 2004), pp. 121–152

Equivalent polynomial eigenvalue problem

change of variable:

$$k_x := k_\mathrm{f} \frac{\gamma + \gamma^{-1}}{2} \quad \Rightarrow \quad k_y = \pm k_\mathrm{f} \frac{\gamma - \gamma^{-1}}{2\mathrm{i}}$$

Inserting into the original **nonlinear eigenvalue problem**:

$$\underline{\underline{F}}(k_x)\underline{q} = \underline{0}\,, \quad \text{with} \quad \underline{\underline{F}}(k_x) = k_x^2 \underline{\underline{A}}_2 + k_x \underline{\underline{A}}_1 + \underline{\underline{A}}_0 + \mathrm{i}\, k_y \underline{\underline{B}}\,,$$

$$\text{where } k_y = \sqrt{k_\mathrm{f}^2 - k_x^2}\,,$$

results in an **equivalent**

polynomial eigenvalue problem:

$$\underline{\underline{P}}(\gamma)\underline{q} = \underline{0} \quad \text{with} \quad \underline{\underline{P}}(\gamma) = \underline{\underline{P}}_4 \gamma^4 + \underline{\underline{P}}_3 \gamma^3 + \underline{\underline{P}}_2 \gamma^2 + \underline{\underline{P}}_1 \gamma + \underline{\underline{P}}_0\,.$$

$\Rightarrow$ **complete** and **unique** problem formulation!

Solving the leaky Lamb problem

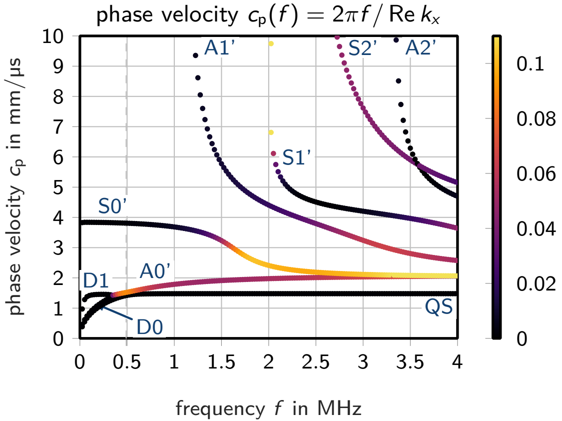
We can reliably solve the leaky Lamb wave problem by performing the following steps:

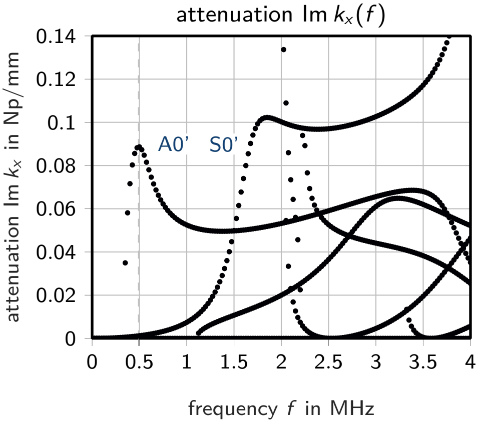
1. Introduce an equivalent **linear eigenvalue problem** (state space formulation)
2. Solve using standard methods
   - $\rightarrow$ eigenvalues γ_n, eigenvector $\underline{q}_n$
3. Compute the wavenumbers $k_x = k_\mathrm{f} \frac{\gamma_n + \gamma_n^{-1}}{2}$ and corresponding $k_y = k_\mathrm{f} \frac{\gamma_n - \gamma_n^{-1}}{2\mathrm{i}}$

Exploit all advantages of modern eigenvalue solvers!

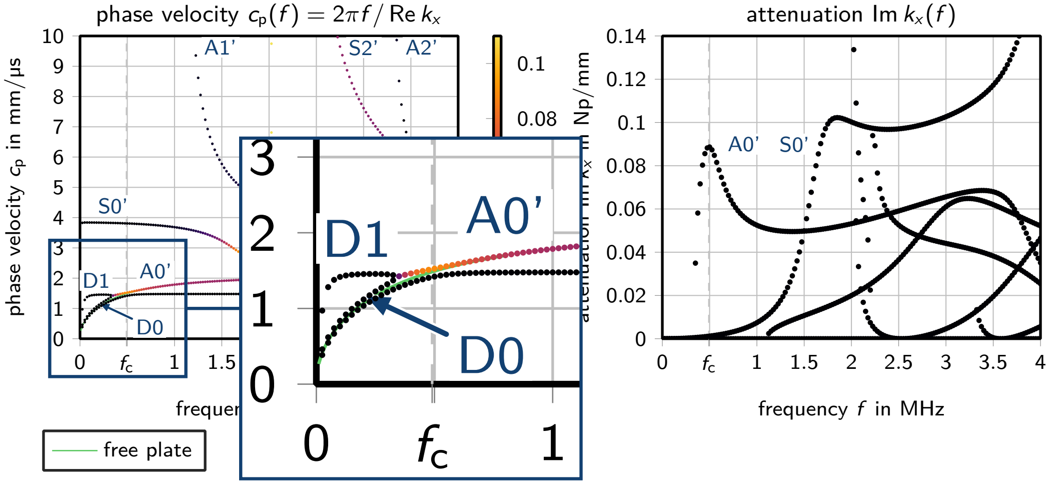
- easy to implement (here: spectral collocation)
- 150 frequency points compute in 1 s on a regular personal computer

Phase velocity and attenuation: 1 mm thick brass plate in contact on one side with water

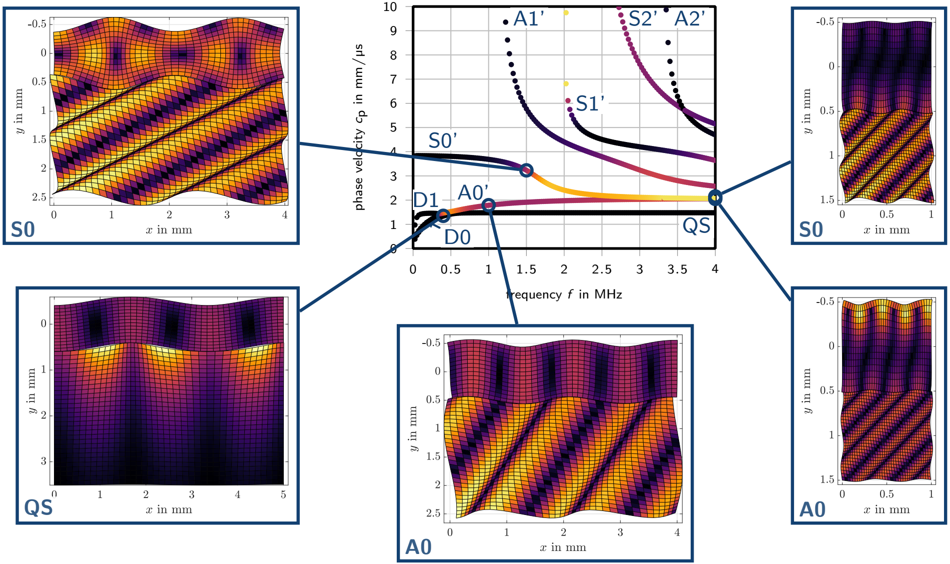




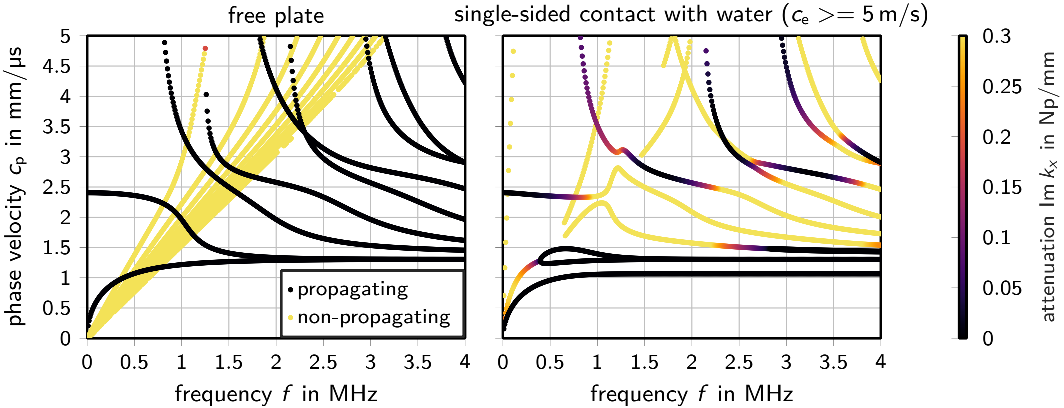
Phase velocity and attenuation: 1 mm thick brass plate in contact on one side with water



Displacement fields: brass plate in contact on one side with water



Free vs. fluid loaded plate: 1 mm thick plexiglass plate



The dispersion curves of a fluid loaded plate can differ substantially from the ones of a free plate.

Conclusions

- We calculated the dispersion characteristics of fluid loaded plates.
- They can be substantially different to the modes of a free plate.
- Avoid root-finding of the characteristic equation.
- Instead: eigenvalue problem with change of variable.

Advantages of the proposed method:

+ linear eigenvalue problem
+ use conventional eigenvalue solvers
  - → **robust**
  - → **efficient**
+ leads to the **full** wavenumber spectrum

Suitable for:

+ viscoelastic, anisotropic, inhomogeneous, and layered plates

Restrictions:

– only plane geometries
– only non-viscous fluids
– not suitable for two different adjacent fluids